# The first meeting

Or rather rendezvous since you usually go to a date whistling, full of hope and heightened expectations. You hope that this relationship will have a future, will be fruitful and you will have pleasant and memorable experiences. The writers of this book are sure that you have made a good decision. Maple is going to be attractive and exciting but certainly not easy. If you only want to flirt with it you had better give it up now.

This chapter aims to convey the first impressions. The tasks to be solved are not trivial but they are easy enough to achieve the target: the acquirement of Maple. While getting to know the input methods of the instructions, the basic Maple data types, the simpler call forms of several methods, you unconsciously start to gain experience of the basic properties of the generic target computer-algebra systems. The technical sheet, which is at the end of this chapter, presents how Maple displays numbers.

## 1.1 The roots of the polynomials

*Prove that if the four real roots of a fourth degree polynomial form an arithmetical sequence, then the same is true for the roots of its derivative.*

The roots are $a$, $a+d$, $a+2\,d$, $a+3\,d$, and enter the polynomial $p$ as well by typing the character sequence

```
p:=(x-a)*(x-(a+d))*(x-(a+2*d))*(x-(a+3*d))
```

after the Maple prompt (>) and press **Enter**.

```
> p := (x-a)·(x-(a+d))·(x-(a+2·d))·(x-(a+3·d))
```

$$p := (x-a)\,(x-a-d)\,(x-a-2\,d)\,(x-a-3\,d) \qquad \textbf{(1)}$$

Let's see what has happened. As we can see, Maple is an interactive system. The communication operates by typing the command chosen – in our case the character sequence `p:=x*(x-d)*(x-2*d)*(x-3*d)` after the Maple prompt. Press **Enter** to close the command. After this the response of the system is displayed on the screen right under the command. In this case this is the repetition of the command in the common mathematical form. Notice that Maple has removed the brackets from certain factors but much more happened in the background of which you are going to hear later.

Notice that when we were typing the command it was not always the typed characters that were displayed on the screen. Namely instead of the asterisk (*) – which is the sign of multiplication in Maple – the dot (.) was displayed. The dot is the sign of multiplication as well but only if we write the formulas by hand. Although the system requires using its syntax when entering the formulas, it tries to please us by converting this form into the common mathematical formula during display.

This behaviour may be disturbing for some users, that is, what we see on the screen is not what we are typing. At this point we can use the **1-D Math** input method instead of **2-D Math** that we have used so far. We have to press **F5** before typing the command. Pressing **F5** makes the cursor turn into a vertical line and the result of the typing is displayed in red.

```
> p:=(x-a)*(x-(a+d))*(x-(a+2*d))*(x-(a+3*d));
```

$$p := (x-a)\,(x-a-d)\,(x-a-2\,d)\,(x-a-3\,d) \qquad \textbf{(2)}$$

This is the **1-D Math** input. Notice that in the case of **1-D Math** the command has to be closed with a semicolon. The advantage of this input is that the typed characters coincide with the ones displayed on the screen. In spite of this you should use the **2-D Math** input whenever it is possible. Since its format is more similar to the common mathematical signs, it is more convenient to use during interactive work. Unfortunately, its usage is limited that is why we need to use **1-D Math** sometimes.

Congratulations if you have managed to display the formulas mentioned above on your screen. You have taken your first step towards acquiring Maple.

But probably you have mistyped at least one of the characters, like this:

```
> p := (x - a) · (x - (a + d) · (x - (a + 2 · d)) · (x - (a + 3· d))
Error, unable to match delimiters
```

$$p := (x-a)\cdot(x-(a+d)\cdot(x-(a+2\cdot d))\cdot(x-(a+3\cdot d))$$

Or like this:

```
> p := (x - a) · (x - (a + d)) · (x - (a +· d)) · (x - (a + 3· d))
Error, invalid product/quotient
```

$$p := (x-a)\cdot(x-(a+d))\cdot(x-(a+\cdot d))\cdot(x-(a+3\cdot d))$$

It is not a serious problem so you do not have to worry about the error messages of the system. Sometimes – but not in all cases – these warnings are understandable and try to inform the user about the error, which sometimes succeeds. Let's take this example. In the first case the system has problems with the pasting of the delimiters, that is, it cannot match the opening and the closing brackets. If you take a closer look at the formula you can notice that a right bracket is missing from the second factor. So the system is right. Later on it will turn out that the system is always right.

The second error message is about an invalid product/quotient. This would be really incomprehensible if Maple did not frame the character sequence +.d in the middle of the output. Now you can easily notice that two figures are missing between the operations + and *.

As for correction, the usual editing functionalities are available when we enter the input line, that is, with the help of the cursor control keys we can move the cursor to the left and the right. **Delete** and **Backspace** are used for deletion. More resolute readers can try the commands **Copy**, **Cut** and **Paste**. Note that a clipboard is also available since we are in a Windows application. Naturally you do not have to retype the wrong command after you sent it and received the error message. You can return to the wrong line with the cursor control keys or with the mouse and correct it. It is also important to note that the cursor need not be put at the end of the command line after finishing the correction. When we press **Enter**, the cursor is allowed to stand anywhere in the line.

Let's correct the wrong line and repeat the command.

```
> p := (x - a) · (x - (a + d)) · (x - (a + 2 · d)) · (x - (a + 3· d))
```

$$p := (x-a)\,(x-a-d)\,(x-a-2\,d)\,(x-a-3\,d) \qquad \textbf{(3)}$$

We have dealt so much with the input of the first command that we may dimly remember the original task. So far we have created the variable $p$ the value of which is the $x\,(x-d)\,(x-2\,d)\,(x-3\,d)$ expression. We can make sure of this if we query the value of $p$. To do this we only have to type its name and then press **Enter**.

```
> p
```

$$(x-a)\,(x-a-d)\,(x-a-2\,d)\,(x-a-3\,d) \qquad \textbf{(4)}$$

The value of $p$ is a polynomial the roots of which form an arithmetical sequence. Concerning the task in hand, we have to show that the derivative of $p$ inherits these properties of $p$. So we have to calculate the

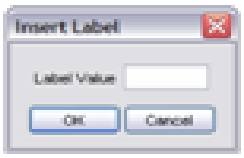
differential by $x$ of the expression. Use the **diff** procedure to do this.

```
> diff(p, x)
```

$$(x-a-d)\,(x-a-2\,d)\,(x-a-3\,d)+(x-a)\,(x-a-2\,d)\,(x-a-3\,d)+(x-a)\,(x-a-d)\,(x-a-3\,d)+(x-a)\,(x-a-d)\,(x-a-2\,d) \quad \textbf{(5)}$$

The first parameter of the **diff** procedure is the expression which we want to differentiate. Its second parameter is the variable according to which the derivation is done. Notice that Maple knows the rules of derivation thus it has created the differential of the polynomial $p$.

Maybe it has escaped your attention but now you have to notice that Maple has numbered the outputs of the given commands, that is, **it has labelled the outputs**. These labels can be used to refer to the different formulas in the text and used in the case of Maple commands as well. To use these labels we need to open a window where we want to put the label reference. We can open the window by pressing **CTRL+L**




and here we can give the label of the formula to which we want to refer in the **Label Value** field. Note that the input window of the labels can be opened by using the **Label** submenu of the **Insert** menu.

Let's see an example. We get the following command by typing the **solve(** character sequence. This time the cursor is on the right side of ( . After this, open the label input window by pressing **CTRL+L** where enter 7 into the **Label Value** field then press **OK**. It makes the system paste the label reference **(5)** in the location where the cursor was. In this way we get the **solve((7)** character sequence which we can complete to the **solve((7)=0,x)** command by typing.

```
> solve((5)=0, x)
```

$$a+\frac{3}{2}\,d,\ \frac{3}{2}\,d+a+\frac{1}{2}\,\sqrt{5}\,d,\ \frac{3}{2}\,d+a-\frac{1}{2}\,\sqrt{5}\,d \quad \textbf{(6)}$$

```
> r := (6)
```

$$r := a+\frac{3}{2}\,d,\ \frac{3}{2}\,d+a+\frac{1}{2}\,\sqrt{5}\,d,\ \frac{3}{2}\,d+a-\frac{1}{2}\,\sqrt{5}\,d \quad \textbf{(7)}$$

The **solve** procedure was used to solve the **(5)** = 0 equation. In the second parameter of the **solve** procedure, we had to give to which variable the solution is to be generated. It is not unnecessary because Maple does not know that we consider x the independent variable of the polynomial while a and d the parameters. Notice that the polynomial **(5)** has three real roots which the system placed in the variable r as a sequence (but separated by commas) according to the second command. We can refer to each root by their sequence indexes.

```
> r[3]; r[1]
```

$$\frac{3}{2}\,d+a-\frac{1}{2}\,\sqrt{5}\,d$$

$$a+\frac{3}{2}\,d \quad \textbf{(8)}$$

Type the name of the variable which contains the sequence. Indicate within square brackets which

element of the sequence you want to get. Notice that the input line mentioned above includes a semicolon, which shows that more commands can be written within a line. Maple executes the commands from the left to the right as if we had given each command in a different line.

The indexing of the sequence elements does not use the common mathematical signs, which is a little bit unusual. But the **2-D Math** input can help us. Type the name of the variable which contains the sequence, that is, $r$ then press underscore ( _ ). As a result the cursor switches to subscript where we can give the number of the preferred element. By pressing **Enter**, Maple executes the command, that is, it displays the preferred index element of the sequence.

> $r_2$

$$\frac{3}{2}\,d + a + \frac{1}{2}\,\sqrt{5}\,d \tag{9}$$

How can the cursor be removed from subscript? Let's take the example that we want the system to calculate the $r_2 - r_1$ difference. During the input of $r_2$, we have to switch to subscript and if we do not exit we can get the following command:

> $r_{2-r_1}$

$$\left(a + \frac{3}{2}\,d,\ \frac{3}{2}\,d + a + \frac{1}{2}\,\sqrt{5}\,d,\ \frac{3}{2}\,d + a - \frac{1}{2}\,\sqrt{5}\,d\right)_{2 - a - \frac{3}{2}\,d} \tag{10}$$

This output is rather surprising. The explanation is that the element of the sequence $r$ was asked which is indexed by $2 - r_1$ . The index was evaluated as $2 - a - \frac{3}{2}\,d$ but since such an element does not exist, Maple listed all the elements of the system. Then it displayed the non-existing index next to the elements, which seems right but it is meaningless for us right now.

The solution is that after typing the index during the input of the element $r_2$, we can exit the subscript by pressing the **right arrow** cursor control key. When you input the next command type the following: r, _, 2, **right arrow**, -, r, _, 1, **Enter**.

> $r_2 - r_1$

$$\frac{1}{2}\,\sqrt{5}\,d \tag{11}$$

If we type the name of the variable which contains the sequence without an index then the system shows the whole sequence in response:

> $r$

$$a + \frac{3}{2}\,d,\ \frac{3}{2}\,d + a + \frac{1}{2}\,\sqrt{5}\,d,\ \frac{3}{2}\,d + a - \frac{1}{2}\,\sqrt{5}\,d \tag{12}$$

These are the roots of the derivatives of polynomial $p$. If we take a closer look at them, we can persuade ourselves that the roots really look like an arithmetical sequence in which the first element of the sequence is $r_3$ and its difference is $\frac{-\sqrt{5}\cdot d}{2}$. While typing the following commands, notice that when you are ready with the subscript, you have to finish the input of the index with the **right arrow** key and continue the typing of the other parts of the command in this way.

> $r_3, r_1, r_2$

$$\frac{3}{2}\,d+a-\frac{1}{2}\,\sqrt{5}\,d,\ a+\frac{3}{2}\,d,\ \frac{3}{2}\,d+a+\frac{1}{2}\,\sqrt{5}\,d \tag{13}$$

> $r_3-r_1,\ r_1-r_2$

$$-\frac{1}{2}\,\sqrt{5}\,d,\ -\frac{1}{2}\,\sqrt{5}\,d \tag{14}$$

The results are convincing. Let us present you one of our students' creative solution who realized that numbers $a$, $b$ and $c$ form a mathematical sequence only if the product of

$$\left(a-\frac{b+c}{2}\right)\left(b-\frac{a+c}{2}\right)\left(c-\frac{a+b}{2}\right)$$

is zero. This statement can be easily seen.

Let's see how it works with the $r$ sequence. But before this you cannot skip practicing the usage of the **2-D Math** input. First, try the input of the $\frac{b+c}{2}$ formula. Type the following: b, +, c, / (it is **SHIFT+6**), 2.

> $b+\frac{c}{2}$

$$b+\frac{1}{2}\,c \tag{15}$$

Notice that after pressing / the **2-D Math** input generated a fraction. It looks nice but you also have to notice that you have not been waiting for this output. If you give it some thought, the procedure is right because fractions are higher priority operations than additions. The solution is that we put the sum $b+c$ between brackets.

> $\frac{(b+c)}{2}$

$$\frac{1}{2}\,b+\frac{1}{2}\,c \tag{16}$$

Now it is working well. But there is a third option for the input of fractions. When using this method, start the fraction by pressing /. This displays a fraction bar and the cursor is blinking in the presently empty numerator of the fraction bar to indicate that the system is waiting for the input of the numerator. After finishing this we can move onto the input of the denominator by pressing the **down arrow cursor control key**. We can switch back to the input of the numerator with the **up arrow cursor control key** and we can exit the input of the fraction with the **right arrow key**. **Enter**, which indicates the end of the command, can be pressed at any positions of the cursor. Feel free to try it

After such a long preparation, we hope that you will succeed in entering the following command the result of which will reassure you that the elements of the $r$ sequence really form an arithmetical sequence.

> $\left(r_1-\frac{r_2+r_3}{2}\right)\cdot\left(r_2-\frac{r_1+r_3}{2}\right)\cdot\left(r_3-\frac{r_1+r_2}{2}\right)$

$$0 \tag{17}$$

>

We have solved the task.

# What have you learnt about Maple?

- Maple commands have two input methods: **2-D Math** and **1-D Math**. You can switch between them by pressing **F5.**
- In **2-D Math** it is not mandatory to close the commands with special characters. Naturally the usage of semicolon is not forbidden. The end of the command is indicated by pressing **Enter**. On the contrary, if the commands are given in **1-D Math** a semicolon is needed before pressing **Enter**.
- More commands can be given within a line. In this case each command is separated by a semicolon.
- The input of the mathematical expressions in **2-D Math** is to be done in the following way.

Asterisk ( * ) is the sign of multiplication but a dot (.) is displayed on the screen.

Underscore ( _ ) makes the cursor switch into subscript from which we can exit with the **right arrow key**.

If we do not want to put the numerator of the fraction containing more terms into extra brackets then it is useful to start the input of the fraction with a / character. In this case the numerator and the denominator can be fixed separately and we can switch between them with the **down** and **up arrow keys**. The input of the fraction is finished with the **right arrow key**.

- The operator of the assignment is the $:=$ character sequence.
- The $diff(f, x)$ command calculates the derivative of the expression $f$ with respect to $x$.
- The $solve(f = 0, x)$ command solves the $f = 0$ equation for the unknown $x$.
- The value of a variable can be queried by typing its name and then pressing **Enter**.
- We can refer to the labels of the outputs by giving the value of the label in the dialogue window called by **CTRL+L**. Maple puts the value of the label in parentheses in the relevant position of the cursor.
- You have become familiar with the data type of the Maple sequence (**expression sequence**) which – as regards its appearance – seems to be the sequence of Maple objects separated by commas. Certain elements of the sequence can be referred to with their sequence indexes and the whole sequence can be referred to with the name of the sequence.